A NOVEL TREATMENT OF OPEN DIELECTRIC WAVEGUIDES

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ABSTRACT

A spectral-domain approach is introduced for the analysis of a general class of open dielectric waveguides. This technique reduces the dimensionality of the space-domain integral equation by use of higher-order boundary conditions. A simple rectangular slab guide is treated as an example and the results are compared to other methods. For the numerical solution of the spectral-domain equations, the method of moments is employed with a Hermite-Gaussian entire-domain basis. It will be seen that only a few basis functions are sufficient to obtain satisfatory results.

I. Introduction

Dielectric structures can play a central role in the submillimeter-wave integrated circuit technology. Development of complex dielectric components relies upon the availability of accurate numerical techniques with high computational efficiency and great versatility. During the past decade, several approximate [1-2] and numerical techniques [3-6] have been proposed for the analysis of two-dimensional dielectric waveguides. The approximate methods developed for optical frequencies become less accurate for submillimeter-wave applications, while the existing numerical methods encounter severe computational problems in view of time and memory requirements as the complexity of the structure increases. Most of these methods perform a fine discretization of the cross section of the waveguide which results in too many unknowns and introduces numerical instabilities. Such limitations make it practically impossible to extend these techniques to three-dimensional structures.

Recently, higher-order boundary conditions have been utilized to formulate planar integral equations for shielded dielectric structures [7]. To this end, an equivalent planar current is defined by averaging the volume polarization current, for which a space-domain planar integral equation is derived. Although this method yields satisfactory results for shielded structures, its application to open waveguides is limited to electrically thin layers, for which only the first few low-order boundary conditions suffice and the inclusion of higher-order ones does not improve the outcome [8].

In this paper, we present a novel technique based on the formulation of a spectral-domain integral equation of reduced dimensionality for open dielectric waveguides of arbitrary dimensions, which can easily be extended to three-dimensional dielectric structures. This technique requires separability of the Fourier transform of the Green's function of the structure with respect to the source and observation coordinates outside the source region. Two vector spectral unknowns are introduced in view of the angular spectrum decomposition of the Green's function. Then using the higher-order boundary conditions on the surface of the dielectric region, coupled integral equations are derived which relate these uknowns through spectral-domain modified Green's functions.

In the following, first the general methodology is developed and then it is applied to to a rectangular dielectric slab waveguide as an example. The resulting spectral-domain integral equation is solved using the method of moments in conjunction with Galerkin's testing. Due to the nature of the spectral unknowns, an entire-domain basis consisting of Hermite-Gaussian orthogonal functions is utilized. Finally, numerical results are disussed and compared with other methods.

II. Theory

In this section, we present the spectral-domain approach for a general class of dielectric waveguides. The geometry of such a waveguide, shown in Fig.1, consists of a dielectric region of rectangular cross section which rests upon a planar layered background structure. An equivalent volume polarization current can be defined as

$$\mathbf{J_p}(x,y) = \begin{cases} jk_0Y_0(\epsilon_r - 1)\mathbf{E}(x,y), & \mid x \mid \leq a, \mid y \mid \leq b \\ 0, & elsewhere \end{cases} \quad (1)$$

where k_0 and Y_0 are the free-space propagation constant and characteristic admittance, respectively, ϵ_r is the relative permittivity of the dielectric region and $\mathbf{E}$ is the electric field inside this region. The uniformity of the structure along the x-axis prompts the use of the Fourier transform with respect

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to the spatial variable x. The Fourier transform of the radiated electric field is then given by

$$\tilde{\mathbf{E}}^{s}(k_x,y)=\int_{-b}^{b}\bar{\bar{\tilde{\mathbf{G}}}}_{\mathbf{e}}(k_x,y\mid y').\tilde{\mathbf{J}}_{\mathbf{p}}(k_x,y')dy' \quad (2)$$

where $\bar{\bar{\tilde{\mathbf{G}}}}_{\mathbf{e}}(k_x,y\mid y')$ and $\tilde{\mathbf{J}}_{\mathbf{p}}(k_x,y')$ are the Fourier transforms of the dyadic Green's function of the background structure and the polarization current, respectively.

For a large class of planar, layered substrate structures, the spectral-domain dyadic Green's function can be separated with respect to y and y' outside the source region, i.e., $\mid y\mid\geq b$. For the geometries of the type shown in Fig.1, the Fourier transform of the Green's function is in the following form:

$$\bar{\bar{\tilde{\mathbf{G}}}}_{\mathbf{e}}(k_x,y\mid y')=\bar{\bar{\mathbf{\Phi}}}_1^{\pm}(k_x,y)e^{\gamma_1 y'}+\bar{\bar{\mathbf{\Phi}}}_2^{\pm}(k_x,y)e^{-\gamma_1 y'} \quad (3)$$

where $\gamma_1^2=k_x^2+k_s^2-k_0^2$, k_s is the propagation constant of the waveguide along the z-axis, the + sign indicates $y\geq b$, and the − sign indicates $y\leq -b$. At this point, we define two vector, spectral, planar unknowns, which have the dimensionality of surface currents, by the following line integrals:

$$\tilde{\mathbf{J}}_{\mathbf{s}_{1,2}}(k_x)=\int_{-b}^{b}e^{-\gamma_1(b\mp y')}\tilde{\mathbf{J}}_{\mathbf{p}}(k_x,y')dy' \quad (4)$$

In view of (4), equation (2) reduces to the form:

$$\tilde{\mathbf{E}}^{s}(k_x,y)=\left[\bar{\bar{\mathbf{\Phi}}}_1^{\pm}(k_x,y).\tilde{\mathbf{J}}_{\mathbf{s1}}(k_x)+\bar{\bar{\mathbf{\Phi}}}_2^{\pm}(k_x,y).\tilde{\mathbf{J}}_{\mathbf{s2}}(k_x)\right]e^{\gamma_1 b} \quad (5)$$

The field inside the dielectric region can then be expanded in a Taylor series about its upper or lower boundaries in the following way:

$$\tilde{\mathbf{E}}(k_x,y)=\sum_{n=0}^{\infty}\frac{(y\mp b)^n}{n!}\frac{\partial^n\tilde{\mathbf{E}}(k_x,\pm b)}{\partial y^n} \quad (6)$$

The higher-order derivatives in (6) can be related to the electric field and its derivatives just outside the boundaries of the dielectric region using the higher-order boundary conditions. In the dyadic form, one can write

$$\frac{\partial^n\tilde{\mathbf{E}}(k_x,\pm b^{\mp})}{\partial y^n}=\bar{\bar{\mathcal{L}}}_n.\tilde{\mathbf{E}}^{s}(k_x,\pm b^{\pm}) \quad (7)$$

where $\bar{\bar{\mathcal{L}}}_n$ is a tensor whose elements are functions of k_x and k_s. Finally, in view of (1),(6) and (7), (4) reduces to the following coupled linear equations:

$$\begin{aligned}\tilde{\mathbf{J}}_{\mathbf{s1}}(k_x)&=\int_{-\infty}^{\infty}\left[\bar{\bar{\tilde{\mathbf{G}}}}_{11}'(k_x\mid\xi).\tilde{\mathbf{J}}_{\mathbf{s1}}(\xi)+\bar{\bar{\tilde{\mathbf{G}}}}_{12}'(k_x\mid\xi)\tilde{\mathbf{J}}_{\mathbf{s2}}(\xi)\right]d\xi\\ \tilde{\mathbf{J}}_{\mathbf{s2}}(k_x)&=\int_{-\infty}^{\infty}\left[\bar{\bar{\tilde{\mathbf{G}}}}_{21}'(k_x\mid\xi).\tilde{\mathbf{J}}_{\mathbf{s1}}(\xi)+\bar{\bar{\tilde{\mathbf{G}}}}_{22}'(k_x\mid\xi).\tilde{\mathbf{J}}_{\mathbf{s2}}(\xi)\right]d\xi\end{aligned} \quad (8)$$

where $\bar{\bar{\tilde{\mathbf{G}}}}_{\mathbf{kl}}'(k_x\mid\xi)$ are modified spectral-domain Green's functions:

$$\begin{aligned}\bar{\bar{\tilde{\mathbf{G}}}}_{1l}'(k_x\mid\xi)&=K\sum_{n=0}^{\infty}(-1)^n I_n(\gamma_1)\bar{\bar{\mathcal{L}}}_n.\bar{\bar{\mathbf{\Phi}}}_l^{+}(\xi,b)\,e^{(\gamma_1'-\gamma_1)b}\mathcal{S}(k_x,\xi)\\ \bar{\bar{\tilde{\mathbf{G}}}}_{2l}'(k_x\mid\xi)&=K\sum_{n=0}^{\infty}I_n(\gamma_1)\bar{\bar{\mathcal{L}}}_n.\bar{\bar{\mathbf{\Phi}}}_l^{-}(\xi,-b)\,e^{(\gamma_1'-\gamma_1)b}\mathcal{S}(k_x,\xi)\end{aligned} \quad (9)$$

with $l=1,2$, $K=jk_0Y_0(\epsilon_r-1)$, $I_n(\gamma_1)$ given by a simple integral which can be evaluated analytically:

$$I_n(\gamma_1)=\int_{-b}^{b}\frac{(b-y)^n}{n!}e^{\gamma_1 y}dy$$

and $\mathcal{S}(k_x,\xi)=\sin[(k_x-\xi)a]/[\pi(k_x-\xi)]$.

It should be noted that due to the rigor of the analysis, equations (8) yield exact relations between $\tilde{\mathbf{J}}_{\mathbf{s1}}$ and $\tilde{\mathbf{J}}_{\mathbf{s2}}$. In the case of a structure with infinite extent along the x-axis, these equations reduce to the exact eigenvalue equations.

Now in order to examine the validity of the general technique presented above, a simple geometry is considered which consists of a rectangular dielectric slab waveguide of width $2a$ and thickness $2b$ immersed in the free space. In this case, the Green's function of the background structure is that of the free space, while the symmetry of the problem along the y-axis decouples the two equations for the two planar unknowns $\tilde{\mathbf{J}}_{\mathbf{s}_{1,2}}$. However, both of the two equations have the same eigenvalues and as a result, only one of them is sufficient for the solution of the eigenvalue problem which is in the following form:

$$\tilde{\mathbf{J}}_{\mathbf{s}}(k_x)=\int_{-\infty}^{\infty}\bar{\bar{\tilde{\mathbf{G}}}}'(k_x\mid\xi).\tilde{\mathbf{J}}_{\mathbf{s}}(\xi)d\xi \quad (10)$$

Note that in this case, the tensor $\bar{\bar{\mathcal{L}}}_n$, introduced above in conjunction with the higher-order boundary conditions, reduces to the following simple form:

$$\bar{\bar{\mathcal{L}}}_{2m}=\gamma_2^{2m}\begin{pmatrix}1&0&0\\0&1/\epsilon_r&0\\0&0&1\end{pmatrix}$$

and

$$\bar{\bar{\mathcal{L}}}_{2m+1}=\gamma_2^{2m}\begin{pmatrix}\mp\gamma_1&jk_x(1-1/\epsilon_r)&0\\0&\mp\gamma_1&0\\0&jk_s(1-1/\epsilon_r)&\mp\gamma_1\end{pmatrix}$$

for even and odd orders, respectively, and $\gamma_2^2=k_x^2+k_s^2-\epsilon_r k_0^2$. Carrying out the infinite summations using known expansions, closed-form expressions for the modified Green's function are obtained. Similar simplifications apply to the more general problem of Fig.1.

III. Numerical Results and Discussion

The integral equation (10) is solved numerically by use of the method of moments in the spectral domain. The choice of the basis functions is dictated by the inverse Fourier transform of $\tilde{\mathbf{J}}_s(k_x)$, shown in Fig.2, which corresponds to a uniform polarization current over $| x | \leq a, | y | \leq b$, with the typical values of $2b = 0.3578\lambda_0, a/b = 4$, and $k_s = 1.26k_0$. Considering the variation of $\mathbf{J}_s(x)$ in the space domain, Hermite-Gaussian functions are chosen as an expansion basis. These functions are given by

$$\phi_n(x) = e^{-\frac{1}{2}(\frac{x}{x_0})^2} H_n(\frac{x}{x_0}) \tag{11}$$

where $H_n(x)$ are the Hermite polynomials and x_0 is a parameter. The Hermite-Gaussian functions have the interesting property that their Fourier transforms are also Hermite-Gaussian of the same order and they form a complete orthogonal system for square-integrable functions over $(-\infty, \infty)$. Thus, the spectral planar unknowns defined previously can be approximated by the following sum:

$$\tilde{J}_{si}(k_x) = \sum_{n=0}^{N} a_{in}\tilde{\phi}_{in}(k_x), \qquad i = x, y, z \tag{12}$$

It turns out that the optimum value of x_0, which minimizes the mean-square error for the approximation of $\tilde{\mathbf{J}}_s(k_x)$, is a function of a, b (the bounds of the dielectric region), N and k_s, but it is not sensitive to the functional form of the polarization current. Therefore, given N and k_s, one can determine the value of x_0 from an examination of a polarization current with uniform distribution over $| x | \leq a, | y | \leq b$.

In view of (13) and with the use of Galerkin's testing, equation (10) reduces to the following matrix equation:

$$\sum_{n=0}^{N} \bar{\bar{\mathbf{K}}}_{mn} \cdot \mathbf{a}_n = 0, \qquad m = 0, \ldots, N \tag{13}$$

where

$$K_{mn}^{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{G}'_{ij}(k_x \mid \xi)\tilde{\phi}^{*}_{im}(k_x)\tilde{\phi}_{jn}(\xi)dk_x d\xi - c_n\delta_{mn}\delta_{ij} \tag{14}$$

with $i, j = x, y, z$, $m, n = 0, \ldots, N$, δ's being the Kronecker delta, and $c_n = \int_{-\infty}^{\infty} \tilde{\phi}^{*}_{im}(k_x)\tilde{\phi}_{jn}(k_x)dk_x$. The propagation constant k_s of the waveguide is found by solving the equation $det[\bar{\bar{\mathbf{K}}}] = 0$.

The spectral-domain technique presented above has been applied to slab waveguides with a variety of parameters. Figs. 3 and 4 show the variation of the normalized propagation constants for the dominant E_{11}^{x} and E_{11}^{y} modes of a rectangular dielectric slab, with $\epsilon_r = 2.25$ and $R = a/b = 4$, versus the normalized guide thickness defined as $B = 4(b/\lambda_0)\sqrt{\epsilon_r - 1}$, respectively. The numerical results have been compared with those of Marcatili's approximate method [1]. Excellent agreement is observed for large waveguide dimensions. However, as the dimensions of the dielectric slab shrink, Marcatili's approximation begins to fail as expected, while this technique yields accurate solutions. The use of the Hermite-Gaussian functions provides satisfactory results with only a few basis functions as opposed to other numerical methods which require fine discretization of the guide cross section. Fig.5 shows the convergence of the eigenvalues for the dominant modes versus the number of basis functions. Additional results including higher-order modes and slabs of different dimensions and dielectric constants will be presented and discussed.

IV. Conclusion

We have developed a general spectral-domain technique which reduces the dimensionality of the space-domain integral equation for a large class of open dielectric waveguides. This technique has the merit of easy extension to three-dimensional structures. As an example, a simple rectangular slab guide was considered and the corresponding spectral-domain modified Green's function was derived. To solve the resulting equations, the method of moments has been employed in the spectral domain using entire-domain Hermite-Gaussian functions. The results show good agreement with other methods, and only a few basis functions are adequate to achieve quick convergence.

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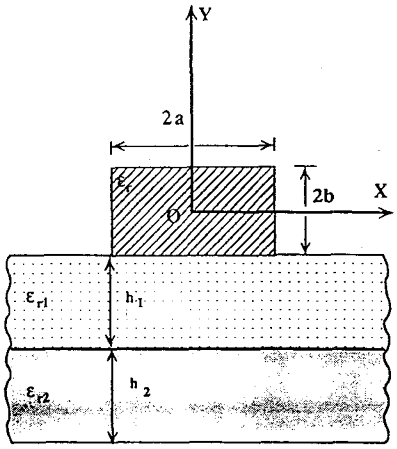

Fig. 1. Ridge dielectric waveguide.

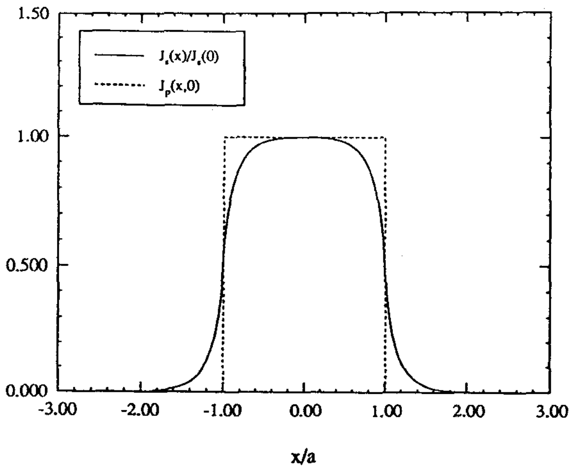

Fig. 2. Inverse Fourier transform of $\tilde{J}_s(k_x)$ corresponding to a uniform volume polarization current.

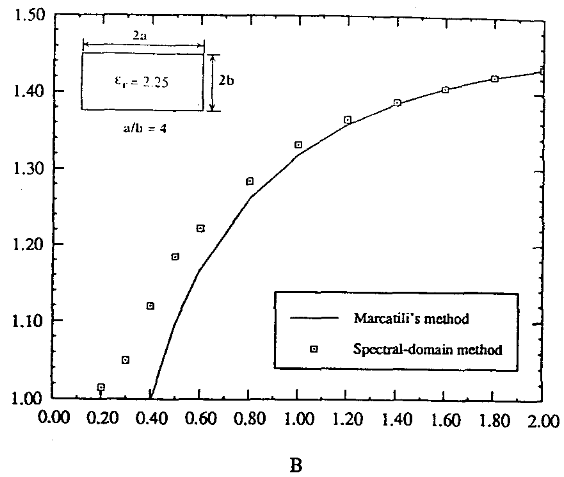

Fig. 3. Normalized propagation constant (k_z/k_0) for the dominant E^x_{11} mode as a function of normalized thickness $B = 4(b/\lambda_0)\sqrt{\epsilon_r - 1}$.

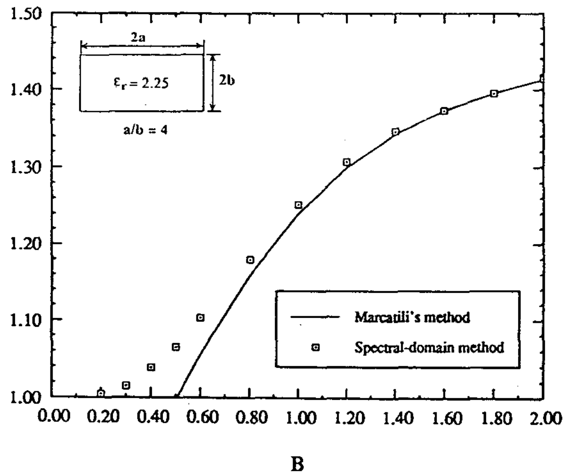

Fig. 4. Normalized propagation constant (k_z/k_0) for the dominant E^y_{11} mode as a function of normalized thickness $B = 4(b/\lambda_0)\sqrt{\epsilon_r - 1}$.

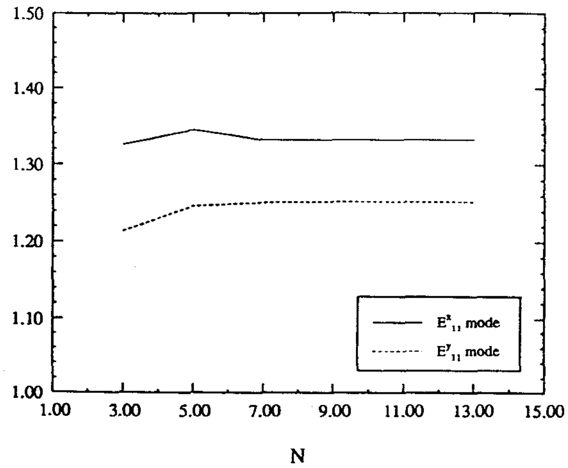

Fig. 5. Convergence of normalized propagation constant vs. the number of basis functions.